Course 2E01 2018 (SF Engineers & MSISS & MEMS)

Sheet 8

Due: at the end of the tutorial

Exercise 1

Calculate the coordinates of $\mathbf{v} = (-3, 2, -1)$ relative to the orthogonal basis

$$\{(2,0,0), (0,2,-3), (0,-3,-2)\}:$$

- (i) with respect to the standard dot product;
- (ii) with respect to the inner product $\langle \mathbf{u}, \mathbf{v} \rangle = u_1 v_1 + 4u_2 v_2 + 4u_3 v_3$ (check that the given basis is still orthogonal with respect to this inner product).

Exercise 2

Find the orthogonal projection of the vector \mathbf{v} onto the plane spanned by the orthogonal basis $\{\mathbf{u}_1, \mathbf{u}_2\}$ (with respect to the standard dot product), where

$$\mathbf{u}_1 = (1, 2, 0), \quad \mathbf{u}_2 = (-2, 1, -2),$$

and

(i)
$$\mathbf{v} = (1, 0, 1);$$

(ii) $\mathbf{v} = (1, 1, 1).$

Exercise 3

Use the Gram-Schmidt process to transform the given basis into orthogonal one:

- (i) $\mathbf{u}_1 = (-1, 0), \, \mathbf{u}_2 = (1, -3);$
- (ii) $\mathbf{u}_1 = (1, 0, -1), \, \mathbf{u}_2 = (1, 0, 0), \, \mathbf{u}_3 = (2, 1, 0).$