October 15, 2018 Lecturer Dmitri Zaitsev

Course 2E01 2018 (SF Engineers & MSISS & MEMS)

S h e e t 5

Due: at the end of the tutorial

Exercise 1

Which of the following sets of vectors are bases for the corresponding space $\mathbb{R}^n$? (The dimension n should be clear from the length of vectors.)

(i) $(1, -1)$;

(ii) $(1, 0)$, $(1, 2)$;

(iii) $(1, 1)$, $(-2, -2)$;

(iv) $(1, 1)$, $(15, 222)$, $(-1, -1)$;

(v) $(1, -1, 2, 0)$, $(1, 1, 5, -3)$, $(1, -1, 2, 1)$;

(vi) $(1, 0, 1)$, $(1, 1, 0)$, $(2, 1, 0)$.

Exercise 2

Find the coordinates of the vector $\mathbf{v}$ with respect to the basis $\mathbf{v}_1, \ldots, \mathbf{v}_n$ (i.e. the coefficients $k_1, \ldots, k_n$ in the representation $\mathbf{v} = k_1\mathbf{v}_1 + \cdots + k_n\mathbf{v}_n$):

(i) $\mathbf{v} = (2, 1)$, $\mathbf{v}_1 = (1, -1)$, $\mathbf{v}_2 = (1, -2)$;

(ii) $\mathbf{v} = (1, -3, 2)$, $\mathbf{v}_1 = (1, 1, 0)$, $\mathbf{v}_2 = (1, 0, 1)$, $\mathbf{v}_3 = (0, 1, 1)$;

(iii) $\mathbf{v} = (1, 1, 1, 0)$, $\mathbf{v}_1 = (1, 0, 1, 0)$, $\mathbf{v}_2 = (1, 1, 0, 0)$, $\mathbf{v}_3 = (0, 0, 2, 0)$, $\mathbf{v}_4 = (1, 0, 0, -1)$.

Exercise 3

Write the general solution of the system as a sum of its partial solution and a linear combination of basis vectors of the associated homogenous system:

(i)

$$\begin{cases} x + y + t = 1 \\ -z + 2t = 3 \end{cases};$$

(ii)

$$\begin{cases} x_4 - x_3 = -1 \\ x_3 - x_2 = 1 \\ x_2 - x_1 = 1 \end{cases};$$

(iii)

$$x_1 - x_2 + x_3 - x_4 = -2.$$