Exercise 1

Find the eigenvalues and corresponding eigenvectors of the following matrix:

\[ A = \begin{pmatrix} -1 & 2 & -1 \\ 0 & 3 & -2 \\ 0 & -6 & 4 \end{pmatrix}. \]

Exercise 2

Find an invertible matrix \( P \) and a diagonal matrix \( D \) diagonalizing \( A \), i.e. satisfying \( P^{-1}AP = D \), where \( A \) is as in Exercise 1.

Exercise 3

Use Exercise 2 to solve the system of ordinary differential equations

\[
\begin{pmatrix} y'_1 \\ y'_2 \\ y'_3 \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix},
\]

where \( A \) is as in Exercise 1.

Exercise 4

Find the Fourier series representation of the function \( f(x) \) for \(-\pi \leq x \leq \pi\), where

\[ f(x) = x + 1. \]