Course 2E01 2017 (SF Engineers & MSISS & MEMS)

S h e e t 7

Due: at the end of the tutorial

Exercise 1

Find the rank and the nullity of the matrix:

(i) $\begin{pmatrix} 2 & -2 & -4 \\ -3 & 3 & 6 \end{pmatrix}$;

(ii) $\begin{pmatrix} 4 & -3 & -6 \\ 2 & 1 & 2 \\ -5 & 0 & 0 \end{pmatrix}$.

Exercise 2

Calculate the length of $\mathbf{u} = (-1, 0, 1)$, the distance between $\mathbf{u}$ and $\mathbf{v} = (0, 1, 1)$ and the angle between $\mathbf{u}$ and $\mathbf{v}$

(i) with respect to the standard dot product;

(ii) with respect to the inner product given by $\langle \mathbf{u}, \mathbf{v} \rangle = 4u_1 v_1 + u_2 v_2 + 2u_3 v_3$.

Exercise 3

Which of the following bases are orthogonal and which are orthonormal with respect to the standard dot product?

(i) $(-1, 1)$, $(0, 3)$;

(ii) $(0, 0, -1)$, $(2, -2, 0)$, $(1, 1, 0)$;

(iii) $(-1, 0, 0)$, $(0, \frac{3}{5}, \frac{4}{5})$, $(0, \frac{4}{5}, -\frac{3}{5})$.