Exercise 1

Find the matrix for the linear transformations $T$ defined by the equations

(i) $w_1 = x_2, \quad w_2 = -x_2 - x_1,$
(ii) $w_1 = x - y, \quad w_2 = z + y, \quad w_3 = -x,$
and by the formulas

(iii) $T(x_1, x_2, x_3) = (x_3 - x_1, x_2, x_1 + 4x_2 + x_3, -2x_1, x_3, x_1).$

Exercise 2

Find $T(x) = Ax$ for the matrix $A$ and the vector $x$ whenever the product makes sense (i.e. the sizes of $A$ and $x$ fit together):

(i) $A = \begin{pmatrix} 0 & -1 & 1 \\ 1 & -1 & 5 \end{pmatrix}, \quad x = \begin{pmatrix} 12 \\ -2 \end{pmatrix},$
(ii) $A = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & -2 \end{pmatrix}, \quad x = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix},$
(iii) $A = \begin{pmatrix} 1 & 2 \\ -1 & 1 \\ 0 & -1 \end{pmatrix}, \quad x = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$

Exercise 3

Use matrix multiplication to find:

(i) the reflection of the vector $(-3, 1)$ about the $y$-axis;
(ii) the reflection of the vector $(-2, 1, 0)$ about the $yz$-plane;
(iii) the orthogonal projection of the vector $(1, -2)$ to the $y$-axis;
(iv) the image of the vector $(-1, 2)$ under rotation through the angle $-\frac{\pi}{4}$ about the origin.
(v) the image of the vector $(2, 0, 1)$ under rotation through the angle $\frac{\pi}{3}$ about $z$-axis.