November 21, 2016 Lecturer Dmitri Zaitsev

**Course 2E01 2016 (SF Engineers & MSISS & MEMS)**

S h e e t 7

Due: at the end of the tutorial

## Exercise 1

Calculate the coordinates of $\mathbf{v} = (1, 2, -3)$ relative to the orthogonal basis

$$\{(3, 0, 0), (0, 1, 2), (0, 4, -2)\} :$$

(i) with respect to the standard dot product;

(ii) with respect to the inner product $\langle \mathbf{u}, \mathbf{v} \rangle = 4u_1v_1 + u_2v_2 + u_3v_3$ (check that the given basis is still orthogonal with respect to this inner product).

## Exercise 2

Find the orthogonal projection of the vector $\mathbf{v} = (1, 1, -1)$ onto the plane spanned by the orthogonal basis (with respect to the standard dot product)

$$\{(2, -4, 0), (-2, -1, 1)\}.$$

## Exercise 3

Use the Gram-Schmidt process to transform the given basis into orthogonal one:

(i) $\mathbf{u}_1 = (-1, 1)$, $\mathbf{u}_2 = (1, 2)$;

(ii) $\mathbf{u}_1 = (1, 0, 1)$, $\mathbf{u}_2 = (0, 0, 1)$, $\mathbf{u}_3 = (0, 2, -1)$.

## Exercise 4

Find the characteristic polynomials of the following matrices:

(ii) $\begin{pmatrix} 0 & -2 \\ -1 & 0 \end{pmatrix}$;

(iii) $\begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & -2 \\ 0 & 0 & 3 \end{pmatrix}$;

(iv) $\begin{pmatrix} 0 & -2 & 1 \\ 1 & 0 & -2 \\ 0 & -2 & 1 \end{pmatrix}$.