Looking for the roots ...

Look among the divisors of -4 — $\pm 1, \pm 2, \pm 4$

Substitutions show: $\lambda = 4$ is a root of $P(\lambda)$

Now divide $P(\lambda)$ by $\lambda - 4$

$$\begin{array}{l|l} \lambda^3 - 8\lambda^2 + 17\lambda - 4 & \lambda - 4 \\ \hline \lambda^3 - 4\lambda^3 & \lambda^2 - 4\lambda + 1 \\ \hline \quad -4\lambda^2 + 17\lambda - 4 & \\ -4(\lambda-4) \quad -4\lambda^2 + 16\lambda & \\ \hline \quad\quad \lambda - 4 & \end{array}$$

$P(\lambda) = (\lambda - 4)(\lambda^2 - 4\lambda + 1)$

Now find the other roots: $\lambda^2 - 4\lambda + 1 = 0$

$\lambda = 2 \pm \sqrt{3}$

We have the e-values : $\lambda_1 = 4$, $\lambda_2 = 2 - \sqrt{3}$, $\lambda_3 = 2 + \sqrt{3}$
(the order plays no role)

Now look for eigenvectors

<u>Case 1.</u> E-vectors corresp. to $\lambda_1 = 4$

$\rightarrow$ solutions of $A\bar{x} = 4\bar{x}$ or

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4x_1 \\ 4x_2 \\ 4x_3 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} -4 & 1 & 0 \\ 0 & -4 & 1 \\ 4 & -17 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

row operations $\rightsquigarrow$ $\begin{pmatrix} -4 & 1 & 0 \\ 0 & -4 & 1 \\ 0 & -16 & 4 \end{pmatrix} \sim \begin{pmatrix} -4 & 1 & 0 \\ 0 & -4 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ $\begin{cases} -4x_1 + x_2 = 0 \\ -4x_2 + x_3 = 0 \end{cases}$

$\begin{cases} x_1 = -\frac{1}{4}x_2 \\ \end{cases}$ $\begin{cases} x_2 = 4x_1 \\ x_3 = 4x_2 \end{cases} \sim \begin{cases} x_2 = 4x_1 \\ x_3 = 16x_1 \end{cases}$

$x_1 = t \quad x_2 = 4x_1 = 4t, \quad x_3 = 4x_2 = 16t$

Eigenspace = set of all solutions

Eigenvector = a solution $\neq 0$

Take $t = 1 \rightsquigarrow$ vector $\bar{v}_1 (x_1, x_2, x_3) = (1, 4, 16)$

is an eigenvector corresp. to $\lambda = 4$.

$\bar{v}_2 = (1, 2-\sqrt{3}, 7-4\sqrt{3})$, $\bar{v}_3 = (1, 2+\sqrt{3}, 7+4\sqrt{3})$

Analogous: $\bar{v}_2$ e-vect. corr to $\lambda_2 = 2-\sqrt{3}$

$\bar{v}_3$ — " — $\lambda_3 = 2+\sqrt{3}$

Diagonalization of matrices (Ch. 7.2) $\longrightarrow$

More special : ~~Fhanoo~~

$A = \begin{pmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{pmatrix}$ ~~$p(\lambda) = \det \begin{pmatrix} +\lambda & 0 & 2 \\ -1 & \lambda-2 & \end{pmatrix}$~~

$p(\lambda) = \det(\lambda I - A) = \lambda^3 - 5\lambda^2 + 8\lambda - 4 = (\lambda-1)(\lambda-2)^2$

$\lambda_{1,2} = 2$ multiple root !

look for e-vectors: $A\bar{x} = \lambda \bar{x}$ or $(\lambda I - A)\bar{x} = 0$

$\begin{pmatrix} 2 & 0 & 2 \\ -1 & 0 & -1 \\ -1 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$x_1 + x_3 = 0 \quad x_2 = t, \quad x_3 = s, \quad x_1 = -s$

$\bar{x} = \begin{pmatrix} -s \\ t \\ s \end{pmatrix} = s \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ Eigenspace is 2-dimensional, has basis

$\bar{v} = (-1, 0, 1)$, $\bar{z}(0, 1, 0) \rightarrow$ eigenbasis corr. to $\lambda = 2$

For $\lambda_3 = 1$, eigensp. is 1-dimensional
$\to$ only 1 vector $\bar{v}_3 = (-2,1,1)$

We have a special basis of $\mathbb{R}^3$:

$\{\bar{v}_1, \bar{v}_2, v_3\}$ consisting ~~only~~ of eigenvectors

If such basis exists, the matrix A can be diagonalized, i.e.

$\exists$ invertible matrix P s.t. $P^{-1}AP$ is diagonal

To find P: Step 1 Find a basis of eigenvectors
$\to \bar{v}_1, \bar{v}_2, \bar{v}_3$
Step 2 Form P having $\bar{v}_1, \bar{v}_2, \bar{v}_3$ as columns:

$$P = \begin{array}{c} \begin{pmatrix} -1 & 0 & -2 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \\ \begin{matrix} \bar{v}_1 & \bar{v}_2 & \bar{v}_3 \end{matrix} \end{array}$$

Then $P^{-1}AP = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$ diagonal!

Indeed: $\cancel{A\bar{v}_1 = \lambda_1 \bar{v}_1}$ $A\Big(^{-1}$

$A\bar{v}_1 = A\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \lambda_1 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \cancel{= 2\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}}$

$A\bar{v}_2 = A\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \lambda_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$A\bar{v}_3 = A\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = \lambda_3 \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$

$$AP = A\begin{pmatrix} -1 & 0 & -2 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1\lambda_1 & 0\cdot\lambda_2 & -2\lambda_3 \\ 0\lambda_1 & 1\cdot\lambda_2 & 1\cdot\lambda_3 \\ 1\cdot\lambda_1 & 0\cdot\lambda_2 & 1\cdot\lambda_3 \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} -1 & 0 & -2 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}}_{A}\begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} = P\begin{pmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \lambda_3 \end{pmatrix}$$

$$\Rightarrow P^{-1}AP = \begin{pmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \lambda_3 \end{pmatrix}$$

Application → Calculating powers

Not every matrix is diagonalizable!

~~$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ -3 & 5 & 2 \end{pmatrix}$ $p(\lambda) = \det(\lambda I - A) =$~~

$= (\lambda-1)(\lambda-2)^2$

~~But: the eigensp for $\lambda_{1,2} = 2$ is~~

~~1-dimensional~~

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \quad \det(\lambda I - A) = \det\begin{vmatrix} \lambda-1 & & \\ & \lambda-1 & \\ & & \lambda-2 \end{vmatrix}$$

$$= (\lambda-1)^2(\lambda-2)$$

$$\lambda_{1,2} = 1 \qquad (1 I - A)(\vec{x}) = 0$$

$$\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0 \qquad x_2 = 0,\ x_3 = 0 \qquad x_1 = s \quad \dim = 1$$

→ no basis!

But: If A has n distinct e-values, it is always diagonalizable

Ch. 11 in 9th ed.

38

Fourier Series (Kreyszig, Ch. 10)

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \quad (*)$$

(Rotating parts of machines, alternating electric currents

Recall: $\cos x$, $\sin x$ are periodic

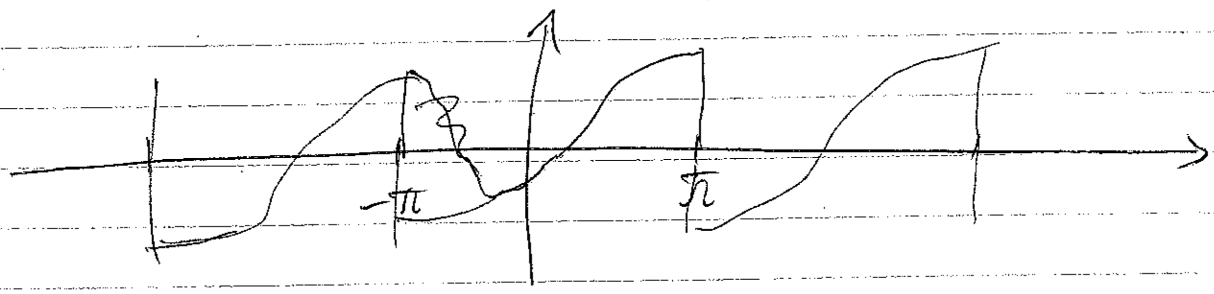
$\cos(x + 2\pi) = \cos x$, $\sin(x + 2\pi) = \sin x$

Also $\cos nx$, $\sin nx$ ~~have~~ are 2π-periodic:

$\cos n(x + 2\pi) = \cos(nx + 2\pi n) = \cos nx$

$\sin n(x + 2\pi) = \dots = \sin nx$

So the series $(*)$ is suitable ~~for f~~ also for f with period 2π: $f(x+2\pi) = f(x)$



Restrict to one period: $[-\pi, \pi]$

Then the coefficients a_n, b_n can be easily determined from $f(x)$ for $x \in [-\pi, \pi]$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\,dx, \quad a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

Euler Formulas

The reason it works is :

$$\int_{-\pi}^{\pi} \cos nx \, dx = \frac{\sin nx}{n}\Big|_{-\pi}^{\pi} = 0 \quad (2\pi\text{-periodic!})$$

$$\int_{-\pi}^{\pi} \sin nx \, dx = -\frac{\cos nx}{n}\Big|_{-\pi}^{\pi} = 0$$

So integrate $(*)$:

$$\int_{-\pi}^{\pi} f(x)\,dx = \underbrace{\int_{-\pi}^{\pi} a_0 \, dx}_{=2\pi a_0} + \sum \Big(a_n \underbrace{\int \cos nx \, dx}_{=0} + b_n \underbrace{\int \sin nx \, dx}_{=0}\Big)$$

Similar : multiply by $\cos mx$ and integrate:

$$\int f(x) \cos mx \, dx = a_0 \underbrace{\int \cos mx \, dx}_{=0} + \sum \Big[a_n \int \cos nx \cos mx \, dx + b_n \int \sin nx \cos mx \, dx \Big]$$

all integrals are zero but one! —

$$a_m \int \cos mx \cdot \cos mx \, dx = \pi \qquad \int \frac{\cos(m+n)x + \cos(m-n)x}{2} \, dx$$

Orthogonality :

~~Main relation:~~

$$\int \cos mx \cos nx \, dx = 0, \quad m \neq n$$

$$\int_{-\pi}^{\pi} \cos mx \cos nx \, dx =$$

$$\int \sin nx \sin mx \, dx = -\int \frac{\cos(n+m)x - \cos(n-m)x}{2} \, dx$$

$$\int \sin nx \cos mx \, dx = \int \frac{\sin(n+m)x - \sin(n-m)x}{2} \, dx$$

Given function $f(x)$, find its Fourier ser. 40

1) $f(x) = 7, \quad -\pi \leq x \leq \pi$

$$a_0 = \frac{1}{2\pi}\int 7\,dx = \frac{1}{2\pi}\cdot 2\pi\, 7 = 7$$

$$a_n = \frac{1}{\pi}\int 7\cos nx\,dx = 0, \quad b_n = 0$$

2) $f(x) = \begin{cases} -k, & -\pi \leq x < 0 \\ k, & 0 \leq x \leq \pi \end{cases}$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi}\int_{-\pi}^{0}(-k)\,dx + \frac{1}{2\pi}\int_{0}^{\pi} k\,dx = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} -k\cos nx\,dx + \int_{0}^{\pi} k\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[-k\frac{\sin nx}{n}\Big|_{-\pi}^{0} + k\frac{\sin nx}{n}\Big|_{0}^{\pi}\right] = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} -k\sin nx\,dx + \int_{0}^{\pi} k\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[+k\frac{\cos nx}{n}\Big|_{-\pi}^{0} - k\frac{\cos nx}{n}\Big|_{0}^{\pi}\right]$$

$$= \frac{k}{n\pi}\left[\cos 0 - \cos(-n\pi) - \cos n\pi + \cos 0\right]$$

$$= \frac{k}{n\pi}(2 - 2\cos n\pi) = \frac{2k}{n\pi}(1-\cos n\pi)$$

$$\cos n\pi = \begin{cases} -1, & n \text{ odd} \\ 1, & n \text{ even} \end{cases} \Rightarrow b_n = \begin{cases} \frac{4k}{n\pi} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

Forier ser: $\sum b_n \cos nx = \frac{4k}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5 \ldots\right.$

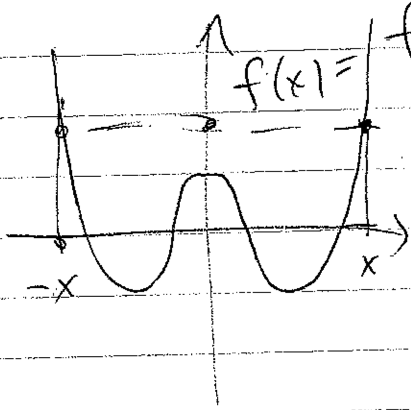
Even and Odd functions

$y = f(x)$ is even if $f(-x) = g(x)$

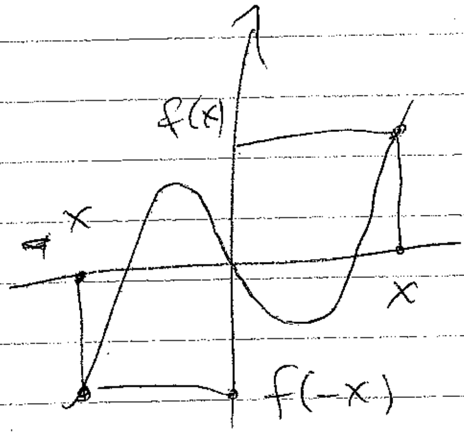
($e.g.$ $\cos[n(-x)] = \cos nx \Rightarrow \cos nx$ even)

$y = f(x)$ is odd if $f(-x) = -f(x)$

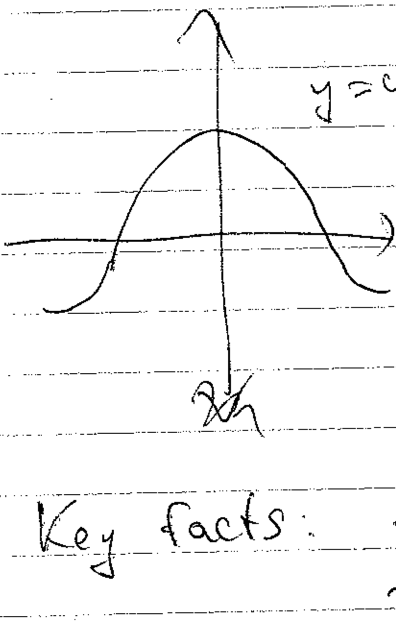
($e.g.$ $\sin n(-x) = -\sin nx \Rightarrow \sin nx$ odd)

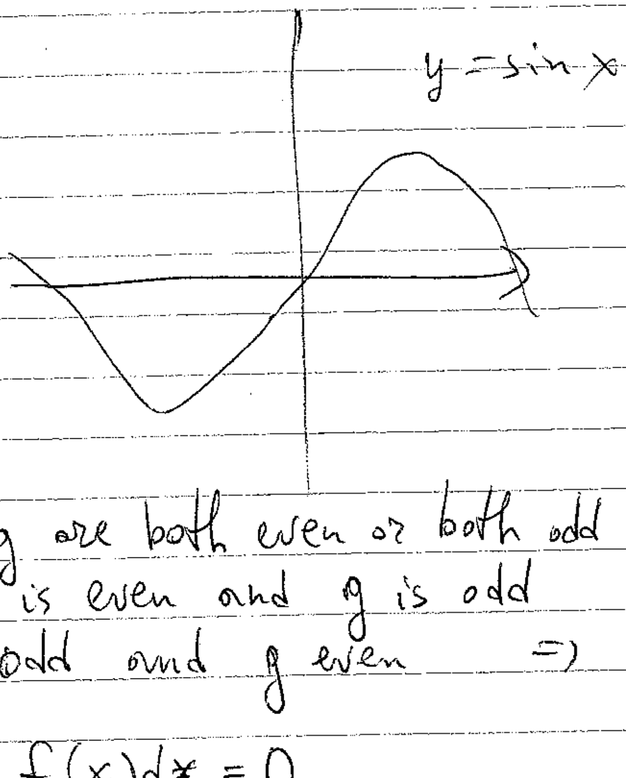


even



odd





Key facts:
- if f, g are both even or both odd $\Rightarrow fg$ even
- if f is even and g is odd or f odd and g even $\Rightarrow fg$ odd

If f is odd, $\int_{-L}^{L} f(x)\,dx = 0$

Thm. (1) The Fourier Series of an even fctn f is

a Fourier cosine series: $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx$

(2) The F. S. of an odd. fctn. f is

a Fourier sine series: $f(x) = \sum_n b_n \sin nx$

The other coefficients are zero!

E.g. if f is even, $\sin nx$ is odd, so $f(x)\sin nx$ odd

$$\Rightarrow b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{f(x)\sin nx}_{\text{odd}}\, dx = 0$$

~~Ex Sawtooth wave $f(x) =$~~

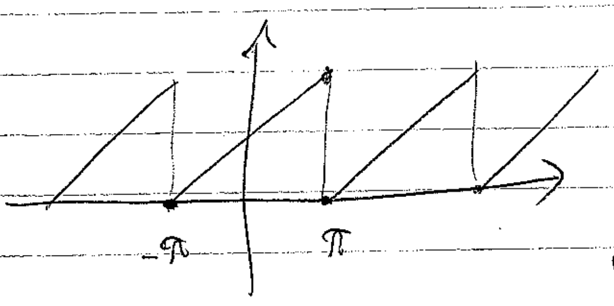
A general f can be always written as $f = g + h$

with g even and h odd!

Indeed, take $g(x) = \frac{f(x)+f(-x)}{2}$, $h(x) = \frac{f(x)-f(-x)}{2}$

Ex. Sawtooth wave $f(x) = x + \pi$ if $-\pi < x < \pi$

and $f(x+2\pi) = f(x)$



We first write $f = \overset{\text{even}}{g} + \overset{\text{odd}}{h}$

$$g(x) = \frac{x+\pi + [(-x)+\pi]}{2} = \pi$$

$$h(x) = \frac{(x+\pi) - (-x+\pi)}{2} = x$$

(43)

For $h(x)=x$ $\quad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x \sin nx \, dx$

Integration by parts: $x \sin nx = uv'$

$u = x \quad v = -\frac{\cos nx}{n}$

$$b_n = \frac{1}{\pi}\left[(uv)\Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} u'v \, dx \right]$$

$$= \frac{1}{\pi}\left[\underbrace{-\frac{x\cos nx}{n}\Big|_{-\pi}^{\pi}}_{-2\cos n\pi} + \underbrace{\int_{-\pi}^{\pi} \frac{\cos nx}{n} dx}_{\substack{=0 \text{ - over the whole period!}}} \right] = -\frac{2}{n}\cos n\pi$$

$\cancel{f(x) = \pi + 2(s}$

$$b_n = \begin{cases} +\frac{2}{n}, & n \text{ odd} \\ -\frac{2}{n}, & n \text{ even} \end{cases}$$

$$f(x) = \pi + 2\left(\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - \dots\right)$$

$p = 2L \qquad f = \sum \cos \frac{n\pi}{L} x$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f \cos\frac{n\pi x}{L}\,dx$$

Application of Fourier Series to Forced Oscillations (Ch. 11.5)

spring

mass m ← position $y(t)$

↓ external force

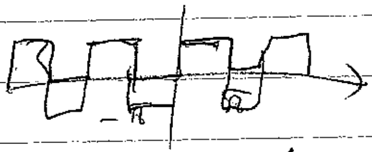
$r(t)$ (time-dependent)

The motion is described by the equation:

$$my'' + cy' + ky = r(t)$$

m mass, c – damping constant, k spring modulus

Ex. r is periodic:



$$r(t) = \begin{cases} 1, & 0 \le t \le \pi \\ -1, & -\pi < t < 0 \end{cases} \qquad m = c = k = 1$$

Method: expand both r and y in Fourier Ser.

$r(t)$ is given → the F.S. is:

$$r(t) = \frac{4}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \dots\right)$$

$$= \frac{4}{\pi} \cancel{\sum_{s=0}^{\infty} \frac{\sin(2s+1)}{2s+1}} \quad \sum_{s=0}^{\infty} \frac{1}{2s+1}\sin(2s+1)$$

$y(t)$ is unknown: $y = a_0 + \sum (a_n \cos nx + b_n \sin nx)$

Substitute into $y'' + y' + y = r(t)$

$$y = a_0 + \sum (a_n \cos nx + b_n \sin nx)$$

$$y' = \sum(-n a_n \sin nx + n b_n \cos nx)$$

$$y'' = \sum(-n^2 a_n \cos nx - n^2 b_n \sin nx)$$

Identify the terms!

	constant	$\cos nx$	$\sin nx$
y''	0	$-n^2 a_n$	$-n^2 b_n$
y'	0	$n b_n$	$-n a_n$
y	a_0	a_n	b_n
r	0	0	$0 \quad \frac{4}{\pi n}$
	$\Downarrow$		$\uparrow$ n even $\quad \uparrow$ n odd
	$a_0 = 0$		

For $\cos nx$: $\quad -n^2 a_n + n b_n + a_n = 0$

For $\sin nx$ $\quad -n^2 b_n - n a_n + b_n = \begin{cases} 0, & n \text{ even} \\ \frac{4}{\pi n}, & n \text{ odd} \end{cases}$

$$n b_n = (n^2 - 1) a_n \qquad b_n = \frac{n^2-1}{n} a_n$$

$$(1-n^2) b_n - n a_n = -\frac{(n^2-1)^2}{n} a_n - n a_n = -\frac{n^4 - 2n^2 + 1 + n^2}{n} a_n$$

$$= -\frac{n^4 - n^2 + 1}{n} a_n = \begin{cases} 0, & n \text{ even} \\ \frac{4}{\pi n}, & n \text{ odd} \end{cases}$$

$$\Rightarrow a_n = \begin{cases} 0, & n \text{ even} \\ -\frac{4}{\pi(n^4 - n^2 + 1)}, & n \text{ odd} \end{cases} \qquad b_n = \frac{n^2-1}{n} a_n$$

$$\Rightarrow y = a_0 + \sum(a_n \cos nx + b_n \sin nx)$$ is/ solution

Fourier Integral and F. Transform (47)

F. Series ~~→ good for~~

$_0 + \sum \left(a_n \cos \frac{n\pi}{L} x + b_n \sin \frac{n\pi}{L} x\right)$ → good for 2L-periodic fctns

For nonperiodic → replace ~~a_n, b_n by~~

~~functions $A(w)$~~, the integer n by variable w and

~~a_n, b_n by~~ $a_n = \frac{1}{\pi} \int_{-L}^{L} f(x) \cos \frac{n\pi}{L} x \, dx$,

$b_n = \frac{1}{\pi} \int_{-L}^{L} f(x) \sin \frac{n\pi}{L} x \, dx$

by functions $A(w) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x) \cos wx \, dx$

$B(w) = \frac{1}{\pi} \int f(x) \sin wx \, dx$

Then ~~$f(x) = \int$~~ F. S. is replaced by integral:

$$f(x) = \int_0^\infty \left(A(w) \cos wx + B(w) \sin wx\right) dw$$

→ Fourier integral representation of f

Fourier Transform

Put cos and sin together!

Euler formula: $\cos x + i\sin x = e^{ix}$

$\cos x - i \sin x = e^{-ix}$

<u>Fourier transform</u> of $f(x)$:

$$\hat{f}(w) := \sqrt{\frac{\pi}{2}}\left(A(w) - i\,B(w)\right)$$

$$\mathcal{F}\{f\}(w) = \sqrt{\frac{\pi}{2}}\left(\frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\,(\cos wx - i \sin wx)\,dx\right)$$

$$\boxed{\hat{f}(w) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\,e^{-iwx}\,dx}$$ Fourier Transf.

$$\mathcal{F}^{-1}\{\hat{f}\}(x) = f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \hat{f}(w)\,e^{iwx}\,dw$$ Inverse F. T.

Similar to Laplace transform but

the inverse tr. is explicit!

$\mathcal{F}\{f'(x)\} = iw\,\mathcal{F}\{f(x)\}$, $\mathcal{F}\{f''(x)\} = (iw)^2\,\mathcal{F}\{f(x)\}$

Convolution: $(f*g)(x) = \int_{-\infty}^{\infty} f(p)\,g(x-p)\,dp$

$\Rightarrow \mathcal{F}\{f*g\} = \sqrt{2\pi}\,\mathcal{F}\{f\}\cdot\mathcal{F}\{g\}$

48

Ex $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 0 \end{cases}$

$$A(w) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(\cancel{x}) \cos w\cancel{x}\, d\cancel{x} = \frac{1}{\pi}\Big(\underbrace{\int_{-\infty}^{-1}}_{0} + \int_{-1}^{1} + \underbrace{\int_{1}^{+\infty}}_{0}\Big) \ldots$$

$$= \frac{1}{\pi}\int_{-1}^{1} \cos w\cancel{x}\, d\cancel{x} = \frac{1}{\pi} \frac{\sin w\cancel{x}}{w}\Big|_{v=-1}^{v=1} = \frac{2\sin w}{\pi w}$$

$$B(w) = \frac{1}{\pi}\int_{-1}^{1} \sin wv\, dv = \frac{1}{\pi} \frac{\cos wv}{w}\Big|_{v=-1}^{v=1} = 0$$

F. Integral: $f(x) = \int_0^\infty \big(A(w) \cos wx + B(w) \sin wx\big) dw$

$$= \int_0^\infty \frac{2\sin w}{\pi w} \cos wx\, dw.$$

Meaning: Each harmonics $\cos wx = h_w(x)$ contributes with

c