Exercise 1

Find the orthogonal projection of the vector \( \mathbf{v} \) onto the plane spanned by the orthogonal basis (with respect to the standard dot product)

\[ \{(1, -2, 0), (-2, -1, 1)\}, \]

where

(i) \( \mathbf{v} = (1, 0, -1) \);
(ii) \( \mathbf{v} = (1, 1, -1) \).

Exercise 2

Use the Gram-Schmidt process to transform the given basis into orthogonal one:

(i) \( \mathbf{u}_1 = (-1, 0), \mathbf{u}_2 = (1, 2) \);
(ii) \( \mathbf{u}_1 = (1, 0, 1), \mathbf{u}_2 = (0, 0, 1), \mathbf{u}_3 = (0, 2, 1) \);

Exercise 3

Find the characteristic polynomials of the following matrices:

(ii) \( \begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix} \);
(iii) \( \begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & -2 \\ 0 & 0 & -3 \end{pmatrix} \);
(iv) \( \begin{pmatrix} 0 & -2 & 1 \\ 1 & 0 & 2 \\ 0 & -2 & 1 \end{pmatrix} \).