Course 2E01 2015 (SF Engineers & MSISS & MEMS)

S h e e t 6

Due: at the end of the tutorial

Exercise 1

Calculate the length of $\mathbf{u} = (-1, 1, 0)$, the distance between $\mathbf{u}$ and $\mathbf{v} = (0, -1, 0)$ and the angle between $\mathbf{u}$ and $\mathbf{v}$

(i) with respect to the standard dot product;

(ii) with respect to the inner product given by $\langle \mathbf{u}, \mathbf{v} \rangle = u_1 v_1 + 3 u_2 v_2 + 2 u_3 v_3$.

Exercise 2

Which of the following bases are orthogonal and which are orthonormal with respect to the standard dot product?

(i) $(2, 0)$, $(0, -1)$;

(ii) $(0, 0, 1)$, $(2, 2, 0)$, $(-1, 1, 0)$;

(iii) $(-1, 0, 0)$, $(0, -\frac{3}{5}, \frac{4}{5})$, $(0, \frac{4}{5}, \frac{3}{5})$.

Exercise 3

Calculate the coordinates of $\mathbf{v} = (1, 2, -3)$ relative to the orthogonal basis

$$\{(3, 0, 0), (0, 2, 4), (0, 4, -2)\} :$$

(i) with respect to the standard dot product;

(ii) with respect to the inner product $\langle \mathbf{u}, \mathbf{v} \rangle = u_1 v_1 + 4 u_2 v_2 + u_3 v_3$ (check that the given basis is still orthogonal with respect to this inner product).