October 12, 2015 Lecturer Dmitri Zaitsev Michaelmas Term 2015

Course 2E01 2015 (SF Engineers & MSISS & MEMS)

S h e e t 2

Due: at the end of the tutorial

Exercise 1

Find the matrix for the linear transformations T defined by the equations

(i) $w_1 = x_1, \quad w_2 = 2x_2 - x_1,$

(ii) $w_1 = -y, \quad w_2 = y + 2z, \quad w_3 = -x,$

and by the formulas

(iii) $T(x_1, x_2) = (x_2, x_1)$,

(iv) $T(x_1, x_2, x_3) = (x_3, x_1 + x_2, x_1 + 4x_2 + x_3, -2x_1, x_3, x_1)$.

Exercise 2

Find $T(\mathbf{x}) = A\mathbf{x}$ for the matrix A and the vector $\mathbf{x}$ whenever the product makes sense (i.e. the sizes of A and $\mathbf{x}$ fit together):

(i) $A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 5 \end{pmatrix}$, $\mathbf{x} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$,

(ii) $A = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & -2 \end{pmatrix}$, $\mathbf{x} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$,

(iii) $A = \begin{pmatrix} 1 & 2 \\ -1 & 4 \\ 0 & -1 \end{pmatrix}$, $\mathbf{x} = \begin{pmatrix} -5 \\ 1 \end{pmatrix}$.

Exercise 3

Use matrix multiplication to find:

(i) the reflection of the vector $(-3, 1)$ about the x-axis;

(ii) the orthogonal projection of the vector $(1, -2)$ to the y-axis;

(iii) the image of the vector $(1, 2)$ under rotation through the angle $\frac{-\pi}{4}$ about the origin.

(v) the image of the vector $(2, 1, 1)$ under rotation through the angle $\frac{\pi}{6}$ about z-axis.