Course 2328 Complex Analysis

Due: Friday, at the end of the lecture

Exercise 1

Find all z such that the sequence (a_n) is convergent:

(i) $a_n := z^n$;

(ii) $a_n := z^n/n$;

(iii) $a_n := z^n/(n!)$;

(iv) $a_n := \mathsf{sin}(z^n)$;

(v) $a_n := \mathsf{Log}(1 - z^n)$ (where Log is the principal value of log);

Exercise 2

For what z does the series converge:

(i) $\sum_n z^{2^n - n}$,

(ii) $\sum_n \frac{z^{n^2}}{n^4}$,

(iii) $\sum_n \frac{1}{z^n - n^2}$.

Exercise 3

Consider the sequence of holomorphic functions $f_n(z) = z - \frac{1}{n}$.

(i) Is the sequence (f_n) converging uniformly on $\mathbb{C}$?

(ii) Is the sequence of squares (f_n^2) converging uniformly on $\mathbb{C}$?

Justify your answer.

Exercise 4

Determine all points $z \in \mathbb{C}$ where the function $f(z)$ has a limit:

(i) $f(z) = \frac{1}{z} + \frac{1}{z^2}$;

(ii) $f(z) = \frac{(\mathsf{Re} z)^2}{z}$;

(iii) $f(z) = \frac{\mathsf{Im} z}{z^3}$;

(iv) $f(z) = \mathsf{Log} z$.

Justify your answer.

Exercise 5

For which $w \in \mathbb{C}$, the power function $f(z) := z^w$ has a continuous branch in the closed upper-half plane $\{\mathsf{Im} z \geq 0\}$?

Exercise 6

Prove or disprove:

(i) A finite union of compact subsets in $\mathbb{C}$ is compact.

(ii) Arbtitrary union of compact subsets in $\mathbb{C}$ is compact.

(iii) A finite union of connected subsets in $\mathbb{C}$ is connected.

(iv) Arbtitrary union of connected subsets in $\mathbb{C}$ is connected.

(v) Arbtitrary union of connected subsets in $\mathbb{C}$ having a nonempty intersection is connected.