Course 2328 Complex Analysis

Due: Friday, at the end of the lecture

Exercise 1
Find all $z$ such that the sequence $(a_n)$ is convergent:

(i) $a_n := z^n$;
(ii) $a_n := z^n/n$;
(iii) $a_n := z^n/(n!)$;
(iv) $a_n := \sin(z^n)$;
(v) $a_n := \log(1 - z^n)$ (where $\log$ is the principal value of $\log$);

Exercise 2
For what $z$ does the series converge:

(i) $\sum_n z^{2n-n}$,
(ii) $\sum_n \frac{z^{n^2}}{n^4}$,
(iii) $\sum_n \frac{1}{z^{n-n^2}}$.

Exercise 3
Consider the sequence of holomorphic functions $f_n(z) = z - \frac{1}{n}$.

(i) Is the sequence $(f_n)$ converging uniformly on $\mathbb{C}$?
(ii) Is the sequence of squares $(f_n^2)$ converging uniformly on $\mathbb{C}$?
Justify your answer.

Exercise 4
Determine all points $z \in \mathbb{C}$ where the function $f(z)$ has a limit:

(i) $f(z) = \frac{1}{z} + \frac{1}{z^2}$;
(ii) $f(z) = \frac{(\text{Re}z)^2}{z}$;
(iii) $f(z) = \frac{\text{Im}z}{z}$;
(iv) $f(z) = \log z$.
Justify your answer.
Exercise 5

For which $w \in \mathbb{C}$, the power function $f(z) := z^w$ has a continuous branch in the closed upper-half plane $\{\text{Im} z \geq 0\}$?

Exercise 6

Prove or disprove:

(i) A finite union of compact subsets in $\mathbb{C}$ is compact.
(ii) Arbitrary union of compact subsets in $\mathbb{C}$ is compact.
(iii) A finite union of connected subsets in $\mathbb{C}$ is connected.
(iv) Arbitrary union of connected subsets in $\mathbb{C}$ is connected.
(v) Arbitrary union of connected subsets in $\mathbb{C}$ having a nonempty intersection is connected.