January 28, 2018 Lecturer Dmitri Zaitsev Hilary Term 2018

Course 2328 Complex Analysis

S h e e t 2

Due: Friday, at the end of the lecture

Exercise 1

Find $\mathsf{log} z$, $\mathsf{Log} z$ and $\mathsf{cos} z$ for

(i) $z = 2i$;

(ii) $z = 1 - i$;

Exercise 2

Prove:

(i) $\mathsf{Im}(iz) = \mathsf{Re} z$, $\mathsf{Re}(iz) = -\mathsf{Im} z$;

(ii) $\mathsf{log}\bar{z} = \overline{\mathsf{log} z}$, $e^{\bar{z}} = \overline{e^{z}}$;

(iii) $\mathsf{cos}\bar{z} = \overline{\mathsf{cos} z}$, $\mathsf{sin}\bar{z} = \overline{\mathsf{sin} z}$.

Exercise 3

(i) Show that $\mathsf{log}(z_1 z_2) = \mathsf{log} z_1 + \mathsf{log} z_2$ as sets.

(ii) Show that $\mathsf{Log}(z_1 z_2) = \mathsf{Log} z_1 + \mathsf{Log} z_2$ provided $-\pi < \mathrm{Arg} z_1 + \mathrm{Arg} z_2 < \pi$.

(iii) Give an example of z_1, z_2 with $\mathsf{Log}(z_1 z_2) \neq \mathsf{Log} z_1 + \mathsf{Log} z_2$.

Exercise 4

Using the definition show:

(i) Finite intersections and arbitrary unions of open sets are open.

(ii) Finite unions and arbitrary intersections of closed sets are closed.

Exercise 5

(i) If Ω is open in $\mathbb{C}$ and $A \subset \mathbb{C}$ any subset, then $\Omega \cap A$ is open in A.

(ii) If $A \subset \mathbb{C}$ is any subset and U is open in A, then there exists Ω open in $\mathbb{C}$ with

$$\Omega \cap A = U.$$

Hint. Use the definition and construct Ω as union of disks.

Exercise 6

Give examples of:

(i) Infinite intersections of open sets that is not open.

(ii) Infinite union of closed sets that is not closed.