March 25, 2017 Lecturer Dmitri Zaitsev Hilary Term 2017

Course 2328 Complex Analysis

Due: Friday, at the end of the lecture

Exercise 1

Let γ be the sum of two line segments connecting -1 with iy and iy with 3, where y is a fixed parameter.

(i) Write an explicit parametrization for γ;

(ii) For every y, evaluate the integrals $\int_\gamma z\,dz$ and $\int_\gamma \bar{z}\,dz$. Which of the integrals is independent of y?

(iii) Use (ii) to show that the conclusion of Cauchy's theorem does not hold for $f(z) = \bar{z}$.

Exercise 2

(i) Calculate $\int_\gamma f(z)\,dz$, where

$$f(z) = \frac{2}{z} - \frac{1}{z^2}$$

and $\gamma(t) = ce^{it}$, $0 \le t \le 2\pi$.

(ii) Use (i) to show that $f(z)$ does not have an antiderivative in its domain of definition.

(iii) Does $f(z) = \frac{1}{z^n}$ have an antiderivative, where $n \ge 2$ is an integer?

(iv) Give an example of an open set Ω, where the function $f(z) = \frac{1}{z(z+1)(z-1)}$ does not have an antiderivative.

Justify your answer.

Exercise 3

Calculate the residues:

(i) $\mathrm{Res}_0 \frac{\sin(z^5) - e^{2z}}{z^9 + z}$;

(ii) $\mathrm{Res}_{-1} \frac{\cos(2\pi z^{-2}) + ze^z}{\sin(2\pi z)}$.

Exercise 4

Evaluate the integrals:

(i) $\int_{-\infty}^{\infty} \frac{x^3 - x}{x^8 + 1}\,dx$.

(ii) $\int_{-\infty}^{\infty} \frac{1 + x + x^2}{x^4 - 2x^2 + 2}\,dx$.