January 20, 2018 Lecturer Dmitri Zaitsev Hilary Term 2017

Course 2328 Complex Analysis

S h e e t 1

Due: Friday, at the end of the lecture

Exercise 1

Find zw, z/w, z^{-101}, for

(i) $z = 1 + i$, $w = 3i + 3$.

(ii) $z = -i$, $w = -2i$.

Exercise 2

Find $\mathsf{log}z$, $\mathsf{Log}z$ and $\sqrt{z}$ for

(i) $z = 2i$;

(ii) $z = 1 - i$;

(iii) $z = 2/(1 - \sqrt{3}i)$.

Exercise 3

Prove:

(i) $\mathsf{Im}(iz) = \mathsf{Re}z$, $\mathsf{Re}(iz) = -\mathsf{Im}z$;

(ii) $\mathsf{log}\bar{z} = \overline{\mathsf{log}z}$, $e^{\bar{z}} = \overline{e^z}$;

(iii) $\mathsf{cos}\bar{z} = \overline{\mathsf{cos}z}$, $\mathsf{sin}\bar{z} = \overline{\mathsf{sin}z}$.

Exercise 4

(i) Show that $\mathsf{log}(z_1 z_2) = \mathsf{log}z_1 + \mathsf{log}z_2$ as sets.

(ii) Show that $\mathsf{Log}(z_1 z_2) = \mathsf{Log}z_1 + \mathsf{Log}z_2$ provided $-\pi < \mathrm{Arg}z_1 + \mathrm{Arg}z_2 < \pi$.

(iii) Give an example of z_1, z_2 with $\mathsf{Log}(z_1 z_2) \neq \mathsf{Log}z_1 + \mathsf{Log}z_2$.

Exercise 5

Sketch the set of points give by the condition:

(i) $0 < |z| < 1$;

(ii) $1 < |2z + i| < 2$;

(iii) $\mathsf{Re}((1 + i)\bar{z}) \geq -1$.