Exercise 1
Let $\gamma$ be the sum of two line segments connecting $-1$ with $iy$ and $iy$ with 1, where $y$ is a fixed parameter.

(i) Write an explicit parametrization for $\gamma$;
(ii) For every $y$, evaluate the integrals $\int_\gamma zdz$ and $\int_\gamma \bar{z}dz$. Which of the integrals is independent of $y$?
(iii) Use (ii) to show that the conclusion of Cauchy’s theorem does not hold for $f(z) = \bar{z}$.

Exercise 2
(i) Calculate $\int_\gamma f(z)dz$, where $f(z) = \frac{1}{z}$ and $\gamma(t) = e^{it}, 0 \leq t \leq 2\pi$, is the unit circle.
(ii) Use (i) to show that $f(z)$ does not have an antiderivative in its domain of definition.
(iii) Does $f(z) = \frac{1}{z^n}$ have an antiderivative, where $n \geq 2$ is an integer?
(iv) Give an example of a domain $\Omega$, where the function $f(z) = \frac{1}{z(z-1)}$ does not have an antiderivative.
Justify your answer.

Exercise 3
Calculate the residues:
(i) $\text{Res}_0 \frac{\sin(z^3) - e^z}{z^3 + z}$;
(ii) $\text{Res}_1 \frac{\cos(2\pi z^2) + z}{e^z - e}$.

Exercise 4
Evaluate the integrals:
(i) $\int_{|z|=2} \frac{e^z}{(z^2 - z)(z+3)}dz$;
(ii) $\int_0^{2\pi} \frac{\sin^2\theta}{2 - \cos\theta} d\theta$;
(iii) $\int_{-\infty}^{\infty} \frac{x - x^2}{x^4 - 2x^2 + 5}dx$. 

Due: at the end of the lecture on Wednesday next week