October 3, 2012 Lecturer Dmitri Zaitsev Michaelmas Term 2012

Course 2325 2012 Complex Analysis I

S h e e t 1

Due: at the end of the lecture on Wednesday of the next week

Exercise 1

Find zw, z/w, z^{100}, for

(i) $z = 1 - i$, $w = 2i + 5$.

(ii) $z = -i$, $w = i + 5$.

Exercise 2

Find $\mathsf{log}z$, $\mathsf{Log}z$ and $\sqrt{z}$ for

(i) $z = -2i$;

(ii) $z = -1 - i$;

(iii) $z = 2/(1 - \sqrt{3}i)$.

Exercise 3

Prove that $\mathsf{Im}(iz) = \mathsf{Re}z$, $\mathsf{Re}(iz) = -\mathsf{Im}z$, $e^{\bar{z}} = \overline{e^z}$, $e^{-z} = \frac{1}{e^z}$.

Exercise 4

(i) Show that $\mathsf{log}(z_1 z_2) = \mathsf{log}z_1 + \mathsf{log}z_2$ as sets.

(ii) Show that $\mathsf{Log}(z_1 z_2) = \mathsf{Log}z_1 + \mathsf{Log}z_2$ provided $-\pi < \mathrm{Arg}z_1 + \mathrm{Arg}z_2 < \pi$.

(iii) Give an example of z_1, z_2 with $\mathsf{Log}(z_1 z_2) \neq \mathsf{Log}z_1 + \mathsf{Log}z_2$.

Exercise 5

Using the definition show:

(i) Finite intersections and arbitrary unions of open sets are open.

(ii) Finite unions and arbitrary intersections of closed sets are closed.

Exercise 6

Construct a branch of $\mathsf{log}\, z$ on the set $\mathbb{C} \setminus \{-iy : y \geq 0\}$. Show that the branch you constructed is indeed continuous.

Exercise 7

Sketch the set of points give by the condition:

(i) $1 < |z| < 3$;

(ii) $1 < |z - 2i| < 2$;

(iii) $\mathsf{Re}((1 - i)\bar{z}) \geq -1$.