October 31, 2012 Lecturer Dmitri Zaitsev Michaelmas Term 2012

Course 2325 2012 Complex Analysis I

Sheet 2

Due: at the end of the lecture on Wednesday of the next week

Exercise 1

For what z does the series converge:

(i) $\sum_n z^{2^n}$,

(ii) $\sum_n \frac{z^{2^n}}{n^2}$,

(iii) $\sum_n \frac{1}{z^n+1}$.

Which are power series? Justify your answer.

Exercise 2

Consider the sequence of holomorphic functions $f_n(z) = z + \frac{1}{n^2}$.

(i) Is the sequence (f_n) converging uniformly on $\mathbb{C}$?

(ii) Is the sequence of squares (f_n^2) converging uniformly on $\mathbb{C}$?

Justify your answer.

Exercise 3

Use the Cauchy-Riemann equations to decide which of the following functions are holomorphic:

$$\mathsf{Re} z, \quad i|z|^3, \quad \bar{z}^2, \quad e^{2z}, \quad e^{\bar{z}}.$$

Exercise 4

Let $f\colon \Omega \to \mathbb{C}$ be holomorphic. Define the new function $\bar{f}$ by $\bar{f}(z) := \overline{f(\bar{z})}$. Show that $\bar{f}$ is holomorhic on the open set $\bar{\Omega} := \{\bar{z} : z \in \Omega\}$.

Exercise 5

Using the Cauchy-Riemann equations, show:

(i) if a holomorphic function f satisfies $\mathsf{Im} f = \text{const}$, then $f = \text{const}$.

(ii) if $f = u + iv$ is holomorphic and $a, b \in \mathbb{C} \setminus \{0\}$ are such that $au + bv = \text{const}$, then again $f = \text{const}$.