November 3, 2010 Lecturer Dmitri Zaitsev Michaelmas Term 2010

Course 2325 2010 Complex Analysis I

S h e e t 2

Due: at the end of the lecture on Wednesday of the next week

Exercise 1

Consider the sequence of holomorphic functions $f_n(z) = z + \frac{1}{n}$.

(i) Is the sequence (f_n) converging uniformly on $\mathbb{C}$?

(ii) Is the sequence of squares (f_n^2) converging uniformly on $\mathbb{C}$?

Justisfy your answer.

Exercise 2

Use the Cauchy-Riemann equations to decide which of the following functions are holomorphic:

$$\mathsf{Im} z, \quad -i|z|^2, \quad \bar{z}^2, \quad e^{z+i}, \quad e^{\bar{z}}.$$

Exercise 3

Let $f\colon \Omega \to \mathbb{C}$ be holomorphic. Define the new function $\bar{f}$ by $\bar{f}(z) := \overline{f(\bar{z})}$. Show that $\bar{f}$ is holomorhic on the open set $\bar{\Omega} := \{\bar{z} : z \in \Omega\}$.

Exercise 4

Using the Cauchy-Riemann equations, show:

(i) if a holomorphic function f satisfies $\mathsf{Re} f = \text{const}$, then $f = \text{const}$.

(ii) if $f = u + iv$ is holomorphic and $a, b \in \mathbb{C} \setminus \{0\}$ are such that $au + bv = \text{const}$, then again $f = \text{const}$.