October 20, 2010 Lecturer Dmitri Zaitsev Michaelmas Term 2010

Course 2325 2010 Complex Analysis I

S h e e t 1

Due: at the end of the lecture on Wednesday of the next week

Exercise 1

Find $\mathsf{log} z$, $\mathsf{Log} z$ and $\sqrt{z}$ for

(i) $z = 2i$;

(ii) $z = 1 - i$;

(iii) $z = 2/(1 + \sqrt{3}i)$.

Exercise 2

Prove that $\mathsf{Im}(iz) = \mathsf{Re}z$, $\mathsf{Re}(iz) = -\mathsf{Im}z$, $e^{\bar{z}} = \overline{e^z}$, $e^{-z} = \frac{1}{e^z}$.

Exercise 3

(i) Show that $\mathsf{log}(z_1 z_2) = \mathsf{log} z_1 + \mathsf{log} z_2$ as sets.

(ii) Show that $\mathsf{Log}(z_1 z_2) = \mathsf{Log} z_1 + \mathsf{Log} z_2$ provided $-\pi < \mathrm{Arg} z_1 + \mathrm{Arg} z_2 < \pi$.

(iii) Give an example of z_1, z_2 with $\mathsf{Log}(z_1 z_2) \neq \mathsf{Log} z_1 + \mathsf{Log} z_2$.

Exercise 4

Using the definition show:

(i) Finite intersections and arbitrary unions of open sets are open.

(ii) Finite unions and arbitrary intersections of closed sets are closed.

Exercise 5

Construct a branch of $\mathsf{log}\, z$ on the set $\mathbb{C} \setminus \{-iy : y \geq 0\}$. Show that the branch you constructed is indeed continuous.

Exercise 6

Sketch the set of points give by the condition:

(i) $1 < |z| < 2$;

(ii) $1 < |z + 2i| < 2$;

(iii) $\mathsf{Re}((1 + i)\bar{z}) \geq 4$.

Exercise 7

Find all z, for which the following identity holds:

$$\frac{1}{4z - z^2} = \frac{1}{4z} + \sum_{n=0}^{\infty} \frac{z^n}{4^{n+2}}.$$