April 11, 2022 Lecturer Dmitri Zaitsev Hilary Term 2022

23204 Introduction to Complex Analysis

S h e e t 9

Exercise 1

For which values α can the Residue theorem be applied to evaluate the integral:

$$\int_0^\infty \frac{x^\alpha}{x^8+1}dx?$$

Evaluate the integral for those values of α.

Exercise 2

For $z_0 \in \mathbb{R}_{>0}$ and the function

$$f(z) = \frac{1}{1+e^z}:$$

(i) characterize subsets of $\mathbb{C}$ where the theorem on power series expansion can be applied to expand f in a power series centered at z_0;

(ii) the same question as in (i) but for the Laurent series expansion of f centered at z_0;

Exercise 3

Prove or disprove:

(i) if a holomorphic function in an open set U achieves a local minimum at a point of U, it is constant in some disk;

(ii) if a holomorphic function in an open set U, that is nowhere zero, achieves a local minimum at a point of U, it is constant in some disk;