March 28, 2022 | Lecturer Dmitri Zaitsev | Hilary Term 2022

23204 Introduction to Complex Analysis

Sheet 7

Exercise 1

For every integer $n \in \mathbb{Z}$, give and example of a point z_0 and a closed path γ with the winding number $W_\gamma(z_0) = n$.

Justify your answer.

Exercise 2

Does there exist a closed path

$$\gamma \colon [a, b] \to \mathbb{C}$$

such that

(i) for every $z_0 \notin \gamma([a, b])$ the winding number $W_\gamma(z_0)$ is even?

(ii) for every $z_0 \notin \gamma([a, b])$ the winding number $W_\gamma(z_0)$ is odd?

Justify your answer.

Exercise 3

Let n be an integer and

$$\gamma_1(t) = e^{int}, \quad t \in [0, 2\pi],$$

and f a holomorphic function in the disk $\Delta_{1+\varepsilon}(0)$, where $\varepsilon > 0$. Show that

$$\int_\gamma f(z)dz = 0.$$

Exercise 4

Consider the paths

$$\gamma_1(t) = a_1 + r_1 e^{it}, \quad \gamma_2(t) = a_2 + r_2 e^{it}, \quad \gamma_3(t) = a_3 + r_3 e^{-it}, \quad t \in [0, 2\pi],$$

where $a_j \in \mathbb{C}$, $r_j \in \mathbb{R}_+$ are given for $j = 1, 2, 3$. Give an example of $a_j \in \mathbb{C}$, $r_j \in \mathbb{R}_+$ such that

$$\sum_j \int_{\gamma_j} \frac{dz}{(z-1)(z+1)} = 0.$$

Justify your answer.