23204 Introduction to Complex Analysis

Sheet 6

Exercise 1

Let $U_1, U_2 \subset \mathbb{C}$ be star-shaped open sets such that $U_1 \cap U_2$ is nonempty and connected. Show that every holomorphic function on $U_1 \cup U_2$ has an antiderivative.

Exercise 2

(i) Give an example of a connected open set $U \subset \mathbb{C}$ which is not star-shaped.

(ii) Give an example of a connected open set $U \subset \mathbb{C}$ which is not star-shaped but for which every holomorphic function on U has an antiderivative.

Justify your answer.

Exercise 3

Set

$$S := \{z \in \mathbb{C} : |\mathsf{Re} z| < 1, |\mathsf{Im} z| < 1\}, \quad cS := \{cz : z \in S\}$$

for $c \in \mathbb{C}$, and consider the open set

$$U := 10S \setminus \overline{S}.$$

Show that for f holomorphic in $\mathbb{C} \setminus \{0\}$:

$$\int_{\partial D_1} f(z)dz = \int_{\partial D_2} f(z)dz$$

(i) for $D_1 = 2S$, $D_2 = 3S$.

(ii) for $D_1 = 4S$, $D_2 = 4e^{i\pi/4}S$.