23204 Introduction to Complex Analysis

Sheet 5

Exercise 1

Let γ be the piecewise affine path representing the oriented boundary of the triangle with vertices

$$-1, 2iy, 1,$$

where y is a fixed parameter.

(i) Write an explicit parametrization for γ;

(ii) For every y, evaluate the integrals $\int_\gamma z\, dz$ and $\int_\gamma \bar{z}\, dz$. Which of the integrals is independent of y?

(iii) Use (ii) to show that the conclusion of Cauchy's theorem does not hold for $f(z) = \bar{z}$.

Exercise 2

Calculate $\int_\gamma f(z)\, dz$, where

(i)

$$f(z) = \frac{2}{z} - \frac{1}{z^2}, \quad \gamma(t) = ce^{it}, \quad 0 \le t \le 2\pi.$$

(ii)

$$f(z) = \frac{2}{z} - \frac{1}{z^2}, \quad \gamma(t) = ce^{it}, \quad 0 \le t \le \pi.$$

(iii)

$$f(z) = \frac{2}{z} - \mathsf{Log} z, \quad \gamma(t) = ce^{it}, \quad -\pi/2 \le t \le \pi/2.$$

Exercise 3

(i) Use 2(i) to show that $f(z)$ does not have an antiderivative in its domain of definition.

(ii) Does $f(z) = \frac{1}{z^n}$ have an antiderivative, where $n \ge 2$ is an integer?

(iv) Give an example of an open set Ω, where the function $f(z) = \frac{1}{z(z-1)^2}$ does not have an antiderivative.

Justify your answer.