February 27, 2022 Lecturer Dmitri Zaitsev Hilary Term 2022

23204 Introduction to Complex Analysis

Sheet 4

Exercise 1

Consider the sequence of holomorphic functions $f_n(z) = z + \frac{1}{n^2}$.

(i) Is the sequence (f_n) converging uniformly on $\mathbb{C}$?

(ii) Is the sequence of squares (f_n^2) converging uniformly on $\mathbb{C}$?

Justisfy your answer.

Exercise 2

Which of the following functions are holomorphic on their domain of definition:

$$z^2 \sin z, \quad (\mathrm{Re} z)^2, \quad z + \bar{z}^2, \quad e^{z\bar{z}}, \quad e^{|z|^2/\bar{z}}?$$

Exercise 3

Using the Cauchy-Riemann equations, show:

(i) if a holomorphic function f satisfies $\mathrm{Re} f = \text{const}$, then $f = \text{const}$.

(ii) if $f = u + iv$ is holomorphic and $a, b \in \mathbb{C} \setminus \{0\}$ are such that $au + bv = \text{const}$, then again $f = \text{const}$.