23204 Introduction to Complex Analysis

S h e e t 3

Exercise 1

Prove or disprove:

(i) A finite union of compact subsets in $\mathbb{C}$ is compact.

(ii) A countable union of compact subsets in $\mathbb{C}$ is compact.

(iii) If S_1, S_2 are connected subsets in $\mathbb{C}$, their union is also connected.

(iv) If S_1, S_2 are connected subsets in $\mathbb{C}$ with $S_1 \cap S_2 \neq \emptyset$, their union is also connected.

Exercise 2

Let $S := \mathbb{C} \setminus [0, +\infty)$.

(i) Construct a branch of $\log z$ on S.

(ii) Find all branches of $\log z$ on S.

Justify your answers.

Exercise 3

Which of the following functions are continuous on $\mathbb{C}$:

(i) $f(z) = \begin{cases} z \text{ if } z \neq 0 \\ 1 \text{ if } z = 0 \end{cases}$

(ii) $f(z) = \begin{cases} |z| \text{ if } |z| \leq 1 \\ \frac{1}{|z|} \text{ if } |z| > 1 \end{cases}$

(iii) $f(z) = \begin{cases} z \text{ if } |z| \leq 1 \\ \frac{1}{z} \text{ if } |z| > 1 \end{cases}$

Justify your answers.