23204 Introduction to Complex Analysis

S h e e t 2

Exercise 1

Which of the following subsets of $\mathbb{C}$ are open, closed or neither open nor closed?

(i) $\{z : \operatorname{Re} z = 0\}$.

(ii) $\{z : \operatorname{Re} z > 0, \operatorname{Im} z = 0\}$.

(iii) $\{z : \operatorname{Im} z \cdot \operatorname{Re} z > 0\}$.

Exercise 2

For each set in Exercise 2 determine:

(i) the interior;

(ii) the boundary;

(iii) the closure.

Exercise 3

For each of the following series, find all z for which the series converges:

(i) $\sum_n z^{n-2^n}$;

(ii) $\sum_n \frac{z^{n^3}}{n^3}$;

(iii) $\sum_n \frac{1}{z^n+1}$ (where z is such that all terms are defined).

Justify your answer.