Course 2318 2011

S h e e t 3

Due: at the end of the tutorial on Thursday of the next week

Exercise 1

Let p_1, p_2, p_3, p_4 be distinct points of $\mathbb{P}^2$ with no 3 collinear.

(i) Prove that there exists unique coordinate system in which the 4 points are $(1,0,0)$, $(0,1,0)$, $(0,0,1)$ and $(1,1,1)$.

(ii) Find all conics passing through $p_1, \ldots, p_4$ and $p_5 = (a,b,c)$.

Exercise 2

Use the parametrization of the cuspidal cubic $C = \{y^2 = x^3\}$ to show that any polynomial vanishing on C is divisible by $y^2 - x^3$.

Exercise 3

Let C be the curve given by $f(x,y) = 0$ and $p = (a,b) \in C$. Assume that the gradient $\nabla f = (f_x, f_y)$ is nonzero at (a,b).

(i) Show that the equation

$$f_x(p)(x-a) + f_y(p)(y-b) = 0$$

defines the unique tangent line to C at p, i.e. the unique line L such that $f|L$ has a multiple root at p.

(ii) Show that the tangent line can be obtained as the limit of the secant line passing through p and another point $q \in C$ as $q \to p$. Hint. Write an equation for the secant, similar to the tangent line equation, where the derivatives are replaced by the corresponding increment ratios.

Exercise 4

Let C be given by $y^2 = x(x-1)(x+1)$.

(i) What is the projectivization $\widetilde{C}$ of C, i.e. the cubic obtained in $\mathbb{P}^2$ by homogenization of the equation for C?

(ii) Find the point at infinity of $\widetilde{C}$.

(iii) Chosing the origin at infinity, find all points of order 2 with respect to the group law, i.e. all points $A \in \widetilde{C}$ with $2A = A + A = 0$. Hint. Use Exercise 3.