Course 214 2009 Complex Analysis)

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Due: at the end of the lecture next week

Exercise 1

Let γ be the sum of two line segments connecting i with x and x with $-i$, where x is on the real line.

(i) Write an explicit parametrization for γ;

(ii) For every x, evaluate the integrals $\int_\gamma z^2\, dz$ and $\int_\gamma \bar{z}^2\, dz$.

Which of these integrals is independent of x?

Exercise 2

Use the theorem on the power series expansion of holomorphic functions to find the radius of convergence of the Taylor series at 0 of the following functions:

(i) $f(z) = \mathsf{Log}(e^z + 1)$ (the principal value);

(ii) $f(z) = \frac{1}{(z+3i)(z-1)\,\mathsf{sin}(z-\frac{\pi}{2})}$;

Justify your anser.

Exercise 3

Evaluate the integrals:

(i) $\int_{|z|=2} \frac{1}{z(z-1)(z-3)}\, dz$;

(ii) $\int_0^{2\pi} \frac{\mathsf{sin}\theta}{2+\mathsf{cos}\theta}\, d\theta$;

(iii) $\int_{-\infty}^{\infty} \frac{x^2}{x^4 - 2x^2 + 2}\, dx$;