March 3, 2009 Lecturer Dmitri Zaitsev Hilary Term 2009

Course 214 2009 Complex Analysis)

Sheet 2

Due: at the end of the lecture first week of Trinity Term

Exercise 1

Prove that $\mathsf{Im}(iz) = \mathsf{Re}z, \quad \mathsf{Re}(iz) = -\mathsf{Im}z, \quad e^{\bar{z}} = \overline{e^z}$.

Exercise 2

(i) Show that $\mathsf{log}(z_1 z_2) = \mathsf{log}z_1 + \mathsf{log}z_2$ as sets.

(ii) Show that $\mathsf{Log}(z_1 z_2) = \mathsf{Log}z_1 + \mathsf{Log}z_2$ provided $-\pi < \mathrm{Arg}z_1 + \mathrm{Arg}z_2 < \pi$.

(iii) Give an example of z_1, z_2 with $\mathsf{Log}(z_1 z_2) \neq \mathsf{Log}z_1 + \mathsf{Log}z_2$.

Exercise 3

Construct a branch of $\mathsf{log}\, z$ on the set $\mathbb{C} \setminus \{iy : y \geq 0\}$.

Exercise 4

Sketch the set of points give by the condition:

(i) $1 < |z| < 2$;

(ii) $1 < |z + i| < 2$;

(iii) $\mathsf{Re}((1 - i)\bar{z}) \geq 2$.

Exercise 5

Find all z, for which the following identity holds:

$$\frac{1}{4z - z^2} = \frac{1}{4z} + \sum_{n=0}^{\infty} \frac{z^n}{4^{n+2}}.$$

Exercise 6

Let $f(z)$ be any branch of $\mathsf{log}z$ defined on an open set. Show that f is holomorphic and $f'(z) = \frac{1}{z}$.

Exercise 7

Using the Cauchy-Riemann equations to decide which of the following functions are holomorphic:

$$\mathsf{Re}z, \quad i|z|^2, \quad \bar{z}^2, \quad e^{z+i}.$$