Exercise 1

(i) For any groups $G_1$ and $G_2$ prove that $G_1 \times \{e\}$ is a normal subgroup in $G_1 \times G_2$.

(ii) Show that the quotient group $(G_1 \times G_2)/(G_1 \times \{e\})$ is isomorphic to $G_2$.

(iii) Prove that the intersection of all normal subgroups in a group is again a normal subgroup.

Exercise 2

Consider the action of $G := \mathbb{Z}$ on $S := \mathbb{Z}_2 \times \mathbb{Z}_4$ given by

$$z \cdot ([a], [b]) := ([z + a], [z + b]).$$

(i) Show that this indeed is a well-defined group action.

(iii) What are the orbits $G \cdot (a, b)$ and what are the stabilizers $G_{(a,b)}$ for $(a, b) \in S$?

Exercise 3

Determine the order of each of the following quotient groups:

(i) $\mathbb{Z}_8/\langle [2] \rangle$

(ii) $\mathbb{Z}_8/\langle [3] \rangle$.

Are these groups cyclic?