

**Course 1214 - Introduction to group theory 2015**

## S h e e t 7

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Due: at the end of the lecture

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**Exercise 1**

- (i) Prove that  $f$  is a homomorphism, where  $f: \mathbb{Z} \rightarrow \mathbb{Z}_n$ ,  $f(a) = [ka]$ , for integer  $k$ ;
- (ii) Prove that there is unique homomorphism  $f: \mathbb{Z}_6 \rightarrow S_3$  with  $f([1]) = (321)$ .

**Exercise 2**

- (i) Prove that a composition of two group homomorphisms is a group homomorphism.
- (ii) Prove that homomorphic image of a cyclic group is cyclic.

**Exercise 3**

Find all homomorphisms:

- (i)  $f: \mathbb{Z}_2 \rightarrow \mathbb{Z}_4$ ,
- (ii)  $f: \mathbb{Z}_2 \rightarrow \mathbb{Z}_5$ .