Exercise 1

(i) Prove that \( f \) is a homomorphism, where \( f: \mathbb{Z} \to \mathbb{Z}_n, f(a) = [ka], \) for integer \( k; \)
(ii) Prove that there is unique homomorphism \( f: \mathbb{Z}_6 \to S_3 \) with \( f([1]) = (321). \)

Exercise 2

(i) Prove that a composition of two group homomorphisms is a group homomorphism.
(ii) Prove that homomorphic image of a cyclic group is cyclic.

Exercise 3

Find all homomorphisms:
(i) \( f: \mathbb{Z}_2 \to \mathbb{Z}_4, \)
(ii) \( f: \mathbb{Z}_2 \to \mathbb{Z}_5. \)