

Course 1214 - Introduction to group theory 2013

S h e e t 5

Due: at the end of the lecture

Exercise 1

Use the Euclidean algorithm to compute the greatest common divisor:

- (i) $\gcd(1001, 33)$
- (ii) $\gcd(56, 126)$
- (iii) $\gcd(234, 2341)$.

Express each greatest common divisor as integer linear combination of the two given integers.

Exercise 2

Use the unique prime factorization to prove:

- (i) $\gcd(ac, bc) = c \gcd(a, b)$ for all integers a, b, c .
- (ii) If $\gcd(a, c) = \gcd(b, c) = 1$, then $\gcd(ab, c) = 1$.

Exercise 3

- (i) Determine cosets in \mathbb{Z}_8 of the subgroup $\langle [4] \rangle$ generated by $[4]$.
- (ii) Determine left and right cosets in S_3 of the subgroup H generated by the cycle (12) .
- (iii) If $H \subset G$ is a subgroup, prove that $gH = H$ if and only if $g \in H$.