

Course 1214 - Introduction to group theory 2015

S h e e t 4

Due: at the end of the lecture

Exercise 1

Find the subgroup generated by the set of permutations (written as cycles):

- (i) $\{(12)\}$;
- (ii) $\{(123)\}$;
- (iii) $\{(12), (123)\}$;
- (iv) $\{(12), (234)\}$;

Exercise 2

Which sets of matrices form group under multiplication:

- (i) $\left\{ A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \in SL_2(\mathbb{Z}) : a_{11} = \pm 1 \right\}$;
- (ii) $\left\{ A \in GL_2(\mathbb{R}) : A \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \right\}$;
- (iii) $\left\{ A \in O_2(\mathbb{Q}) : A \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \right\}$;
- (iv) $\left\{ A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \in U_2 : a_{11}a_{22} = 1 \right\}$.