February 4, 2015 Lecturer Dmitri Zaitsev Hilary Term 2015

Course 1214 - Introduction to group theory 2015

Sheet 3

Due: at the end of the lecture

Exercise 1

For $a = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 3 & 2 & 5 \end{pmatrix}$ and $b = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 1 & 2 & 4 \end{pmatrix}$:

(i) compute ab and a^{-1};

(ii) solve the equation $ax = b$;

(iii) write b as product of transpositions and determine its sign;

Exercise 2

Which groups are cyclic:

(i) the symmetry group S_3;

(ii) the subgroup $n\mathbb{Z} \subset \mathbb{Z}$;

(iii) the additive group $\mathbb{Z}$;

(iv) the group of all translations of $\mathbb{R}$.

(v) the group of all matrices of the form $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$, where $a \in \mathbb{Q}$.

Exercise 3

Write the permutation as product of disjoint cycles and determine its sign:

(i) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 1 & 2 & 4 \end{pmatrix}$;

(ii) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 1 & 6 & 2 & 4 \end{pmatrix}$;

(iii) $(12)(234)(34567)$ (product of overlapping cycles).