January 28, 2015 | Lecturer Dmitri Zaitsev | Hilary Term 2015

Course 1214 - Introduction to group theory 2015

S h e e t 2

Due: at the end of the lecture

Exercise 1

For which binary operations $*$ on the rational numbers $\mathbb{Q}$ there is identity element:

(i) $m * n = mn$;
(ii) $m * n = m + n - 1$;
(iii) $m * n = \frac{m+n}{2}$;
(iv) $m * n = -1$.

Exercise 2

Prove that associativity $(ab)c = a(bc)$ holds automatically whenever one of the elements a, b, c is the identity e.

Exercise 3

Which sets S with operations are groups:

(i) $S = \{-1, 1, 0\}$ with respect to multiplication;
(ii) $S = \{-1, 0, 1\}$ with respect to addition;
(iii) $S = \mathbb{Z} \setminus \{0\}$ with respect to multiplication;
(iv) $S = \{5n : n \in \mathbb{Z}\}$ with respect to addition;
(v) $S = \{5n : n \in \mathbb{Z}\}$ with respect to multiplication;
(vi) $S = \mathbb{Z}$ with respect to subtraction;
(vii) $S = \{(-2)^n : n \in \mathbb{Z}\}$ with respect to multiplication.

Exercise 4

Prove that in any group

(i) $(xy)^{-1} = y^{-1}x^{-1}$;
(ii) identity e is the only solution of the equation $x^2 = x$.

Prove or give a counterexample:

(iii) Is e always the only solution of $x^3 = x$?