

**Course 1214 - Introduction to group theory 2013**

## S h e e t 9

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Due: at the end of the lecture

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**Exercise 1**

- (i) For any groups  $G_1$  and  $G_2$  prove that  $G_1 \times \{e\}$  is a normal subgroup in  $G_1 \times G_2$ .
- (ii) Prove that the intersection of all normal subgroups in a group is again a normal subgroup.

**Exercise 2**

Find all homomorphisms:

- (i)  $f: \mathbb{Z}_2 \rightarrow \mathbb{Z}_4$ ,
- (ii)  $f: \mathbb{Z}_2 \rightarrow \mathbb{Z}_5$ .

**Exercise 3**

Determine the order of each of the following quotient groups:

- (i)  $\mathbb{Z}_8 / \langle [4] \rangle$
- (ii)  $\mathbb{Z}_8 / \langle [3] \rangle$ .

Are these groups cyclic?