Course 1214 - Introduction to group theory 2013

Sheet 8

Due: at the end of the lecture

Exercise 1

(i) Prove that f is a homomorphism, where $f\colon \mathbb{Z} \to \mathbb{Z}_n$, $f(a) = [ka]$, for integer k;

(ii) Prove that there is unique homomorphism $f\colon \mathbb{Z}_6 \to S_3$ with $f([1]) = (123)$.

Exercise 2

(i) Construct an isomorphism between $\mathbb{Z}$ and the group of all numbers 3^n, $n \in \mathbb{Z}$.

(ii) Construct an isomorphism between $\mathbb{Z} \times \mathbb{Z}$ and the group of all numbers $2^m 3^n$, $m, n \in \mathbb{Z}$.

Exercise 3

(i) Prove that a composition of two group homomorphisms is a group homomorphism.

(ii) Prove that homomorphic image of a cyclic group is cyclic.