Course 1214 - Introduction to group theory 2013

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Due: at the end of the lecture

Exercise 1

Use the Euclidean algorithm to compute the greatest common divisor:

(i) $\gcd(101, 33)$

(ii) $\gcd(56, 126)$

(iii) $\gcd(25, 255)$.

Express each greatest common divisor as integer linear combination of the two given integers.

Exercise 2

Use the unique prime factorization to prove:

(i) $\gcd(ac, bc) = c \gcd(a, b)$ for all integers a, b, c.

(ii) If $\gcd(a, c) = \gcd(b, c) = 1$, then $\gcd(ab, c) = 1$.

Exercise 3

(i) Determine cosets in $\mathbb{Z}_8$ of the subgroup $\langle [4] \rangle$ generated by $[4]$.

(ii) Determine left and right cosets in S_3 of the subgroup H generated by the cycle (23).

(iii) If $H \subset G$ is a subgroup, prove that $gH = H$ if and only if $g \in H$.