February 18, 2013 Lecturer Dmitri Zaitsev Hilary Term 2013

Course 1214 - Introduction to group theory 2013

S h e e t 5

Due: at the end of the lecture

Exercise 1

For points (x_1, y_1) and (x_2, y_2) in the plane $\mathbb{R}^2$, determine which are equivalence relations:

(i) $(x_1, y_1) \sim (x_2, y_2)$ if $y_1 = y_2$;

(ii) $(x_1, y_1) \sim (x_2, y_2)$ if $x_1 = x_2$ or $y_1 = y_2$;

(ii) $(x_1, y_1) \sim (x_2, y_2)$ if $x_1 - x_2$ is integer.

For the equivalence relations, determine equivalence classes.

Exercise 2

(i) Prove that if $a|b$ (a divides b) and $b|c$, then $a|c$.

(ii) Prove that if $a|b$ and $b|a$, then $a = \pm b$.

Exercise 3

(i) For each pair a, b, perform the division of a by b with remainder:

$$a = 19,\ b = 5, \quad a = -7,\ b = 5;$$

(ii) Prove that if $m|n$ and $a \equiv b \bmod n$, then $a \equiv b \bmod m$;

(iii) For which n is $25 \equiv 4 \bmod n$?