Course 1214 - Introduction to group theory 2013

S h e e t 4

Due: at the end of the lecture

Exercise 1

Write the permutation as product of disjoint cycles and determine its sign:

(i) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 1 & 5 & 2 \end{pmatrix}$;

(ii) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 1 & 2 & 6 & 4 \end{pmatrix}$;

(iii) $(12)(234)(3456)$ (product of overlapping cycles).

Exercise 2

Find the subgroup generated by the set of permutations (written as cycles):

(i) $\{(12)\}$;

(ii) $\{(123)\}$;

(iii) $\{(12), (123)\}$;

Exercise 3

Which sets of matrices form group under multiplication:

(i) $\{A \in SL_3(\mathbb{Z}) : a_{11} = 1\}$;

(ii) $\{A \in \mathbb{R}^{2 \times 2} : A \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\}$;

(iii) $\{A \in O_2(\mathbb{Q}) : A \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\}$.