Course 1214 - Introduction to group theory 2013

S h e e t 3

Due: at the end of the lecture

Exercise 1

Prove that in any group

(i) $(xy)^{-1} = y^{-1}x^{-1}$;

(ii) identity e is the only solution of the equation $x^2 = x$.

(iii) Is e always the only solution of $x^3 = x$? Prove or give a counterexample.

Exercise 2

For $a = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 3 & 2 & 5 \end{pmatrix}$ and $b = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 2 & 1 \end{pmatrix}$:

(i) compute ab and a^{-1};

(ii) solve the equation $ax = b$;

(iii) write b as product of transpositions and determine its sign;

Exercise 3

Which groups are cyclic:

(i) the symmetry group S_2;

(ii) the subgroup $n\mathbb{Z} \subset \mathbb{Z}$;

(iii) the additive group $\mathbb{Q}$;

(iv) the group of all rotations of $\mathbb{R}^2$.

(v) the group of all matrices of the form $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$, where $a \in \mathbb{Z}$.