Exercise 1
For which binary operations $\ast$ on the rational numbers $\mathbb{Q}$ there is identity element:

(i) $m \ast n = 2mn$;
(ii) $m \ast n = m + n + 1$;
(iii) $m \ast n = \frac{m-n}{2}$;
(iv) $m \ast n = 11$.

Exercise 2
Prove that associativity $(ab)c = a(bc)$ holds automatically whenever one of the elements $a, b, c$ is the identity $e$.

Exercise 3
Which sets $S$ with operations are groups:

(i) $S = \{-1, 1\}$ with respect to multiplication;
(ii) $S = \{-1, 0, 1\}$ with respect to addition;
(iii) $S = \mathbb{Z}^*$ with respect to multiplication ($\mathbb{Z}^* = \mathbb{Z} \setminus \{0\}$);
(iv) $S = \{5n : n \in \mathbb{Z}\}$ with respect to addition;
(v) $S = \{5n : n \in \mathbb{Z}\}$ with respect to multiplication;
(vi) $S = \mathbb{Z}$ with respect to subtraction;
(vii) $S = \{2^n : n \in \mathbb{Z}\}$ with respect to multiplication.