

Course 1214 - Introduction to group theory 2013

S h e e t 2

Due: at the end of the lecture

Exercise 1

For which binary operations $*$ on the rational numbers \mathbb{Q} there is identity element:

- (i) $m * n = 2mn$;
- (ii) $m * n = m + n + 1$;
- (iii) $m * n = \frac{m-n}{2}$;
- (iv) $m * n = 11$.

Exercise 2

Prove that associativity $(ab)c = a(bc)$ holds automatically whenever one of the elements a, b, c is the identity e .

Exercise 3

Which sets S with operations are groups:

- (i) $S = \{-1, 1\}$ with respect to multiplication;
- (ii) $S = \{-1, 0, 1\}$ with respect to addition;
- (iii) $S = \mathbb{Z}^*$ with respect to multiplication ($\mathbb{Z}^* = \mathbb{Z} \setminus \{0\}$);
- (iv) $S = \{5n : n \in \mathbb{Z}\}$ with respect to addition;
- (v) $S = \{5n : n \in \mathbb{Z}\}$ with respect to multiplication;
- (vi) $S = \mathbb{Z}$ with respect to subtraction;
- (vii) $S = \{2^n : n \in \mathbb{Z}\}$ with respect to multiplication.