Exercise 1

How many maps, injective maps, surjective maps and bijective maps $f$ from $A$ to $B$ exist for

(i) $A = \{1\}, B = \{1, 2\}$;

(ii) $A = \{1, 2\}, B = \{1, 2\}$;

(iii) $A = \{1, 2\}, B = \{1, 2, 3\}$.

Exercise 2

Find the inverse map $f^{-1}$ for

(i) $f(x) = -5x$;

(ii) $f(x) = x + 2$;

(iii) $f(x) = e^x$.

Exercise 3

Let $f: S \to T$ be a map and $A, B \subset S$ be two subsets.

(i) Show that $f(A \cup B) = f(A) \cup f(B)$;

(ii) Show that $f(A \cap B) \subset f(A) \cap f(B)$ and illustrate by example that “$\subset$” cannot be replaced by “$=$”.

Exercise 4

Which binary operations $\ast$ on the natural numbers $\mathbb{N}$ are commutative and which are associative:

(i) $m \ast n = mn + 1$;

(ii) $m \ast n = \frac{m-n}{2}$;

(ii) $m \ast n = 55$. 