

**Course 1214 - Introduction to group theory 2013**

## S h e e t 1

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Due: at the end of the lecture

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**Exercise 1**

How many maps, injective maps, surjective maps and bijective maps  $f$  from  $A$  to  $B$  exist for

- (i)  $A = \{1\}$ ,  $B = \{1, 2\}$ ;
- (ii)  $A = \{1, 2\}$ ,  $B = \{1, 2\}$ ;
- (iii)  $A = \{1, 2\}$ ,  $B = \{1, 2, 3\}$ .

**Exercise 2**

Find the inverse map  $f^{-1}$  for

- (i)  $f(x) = -5x$ ;
- (ii)  $f(x) = x + 2$ ;
- (iii)  $f(x) = e^x$ .

**Exercise 3**

Let  $f: S \rightarrow T$  be a map and  $A, B \subset S$  be two subsets.

- (i) Show that  $f(A \cup B) = f(A) \cup f(B)$ ;
- (ii) Show that  $f(A \cap B) \subset f(A) \cap f(B)$  and illustrate by example that “ $\subset$ ” cannot be replaced by “ $=$ ”.

**Exercise 4**

Which binary operations  $*$  on the natural numbers  $\mathbb{N}$  are commutative and which are associative:

- (i)  $m * n = mn + 1$ ;
- (ii)  $m * n = \frac{m-n}{2}$ ;
- (ii)  $m * n = 55$ .