November 19, 2018 Lecturer Dmitri Zaitsev

Course 1213 - Introduction to group theory 2018

Sheet 9

Due: at the end of the tutorial

Exercise 1

(i) Prove that for every integer k,

$$f\colon \mathbb{Z} \to \mathbb{Z}_n, \quad a \mapsto [ka]$$

is a group homomorphism;

(ii) Find all homomorphisms $f\colon \mathbb{Z}_4 \to S_3$ (where S_3 is the symmetric group).

Exercise 2

Find all solutions of the system in $z \in \mathbb{Z}$:

(i)

$$\begin{cases} z \equiv -1 \bmod 5 \\ z \equiv 3 \bmod 6 \\ z \equiv 2 \bmod 8 \end{cases},$$

(ii)

$$\begin{cases} z \equiv 1 \bmod 5 \\ z \equiv 1 \bmod 3 \\ z \equiv 5 \bmod 2 \end{cases}.$$

Exercise 3

(i) For any groups G_1 and G_2 prove that $G_1 \times \{e\}$ and $\{e\} \times G_2$ are normal subgroups in $G_1 \times G_2$.

(ii) Show that the quotient group $(G_1 \times G_2)/(G_1 \times \{e\})$ is isomorphic to G_2.

(iii) Prove that the intersection of any family of normal subgroups in a group is again a normal subgroup.

Exercise 4

Let H be a normal subgroup in a group G, and $f\colon G \to G'$ a group homomorphism. Show that the construction

$$[g] \mapsto f(g)$$

gives a well-defined map $G/H \to G'$ if and only if $H \subset \ker f$.