November 5, 2018 Lecturer Dmitri Zaitsev

Course 1213 - Introduction to group theory 2018

Sheet 7

Due: at the end of the tutorial

Exercise 1

Given a binary relation R, we write $a \sim b$ whenever $(a, b) \in R$. For points (x_1, y_1) and (x_2, y_2) in the plane $\mathbb{R}^2$, determine which are equivalence relations:

(i) $(x_1, y_1) \sim (x_2, y_2)$ if $x_1 + y_1 = x_2 + y_2$;

(ii) $(x_1, y_1) \sim (x_2, y_2)$ if $x_1 = x_2$ or $y_1 = -y_2$;

(ii) $(x_1, y_1) \sim (x_2, y_2)$ if $2y_1 - 2y_2$ is integer.

For the equivalence relations, determine equivalence classes.

Exercise 2

(i) For each pair a, b, perform the division of a by b with remainder:

$$a = -129,\ b = 5, \quad a = 129,\ b = 7;$$

(ii) Prove that if $m|n$ and $a \equiv b \bmod n$, then $a \equiv b \bmod m$;

(iii) For which n is $59 \equiv -1 \bmod n$?

Exercise 3

Prove or disprove:

(i) $(a \equiv 5 \bmod n)$ implies $(a \equiv 5 + n \bmod n)$;

(ii) $(a \equiv 5 \bmod n)$ implies $(a \equiv 5 + a \bmod n)$;

(iii) $(a \equiv b \bmod n)$ implies $(a \equiv n \bmod b)$.