October 8, 2018 Lecturer Dmitri Zaitsev

Course 1213 - Introduction to group theory 2018

S h e e t 4

Due: at the end of the tutorial

Exercise 1

Which H are subgroups of G:

(i) $G = (\mathbb{Z}, +)$, $H = \{0, \pm 1\}$;

(ii) $G = (\mathbb{Q}^*, \cdot)$, $H = \{\pm 1\}$;

(iii) $G = S_3$, $H = \left\{ e, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \right\}$;

(iv) $G = S_3$, $H = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \right\}$;

(iii) $G = S_3$, $H = \left\{ e, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \right\}$.

Exercise 2

Find the subgroup of S_4 generated by the set of permutations:

(i) $\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix} \right\}$;

(ii) $\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \right\}$;

(iii) $\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & 3 \end{pmatrix} \right\}$;

(iv) $\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix} \right\}$.

Exercise 3

Which groups are cyclic:

(i) the symmetry group S_2;

(ii) the subgroup $n\mathbb{Z} \subset \mathbb{Z}$;

(iii) the subgroup generated by $\{1/2, 1/3\}$ in $(\mathbb{Q}, +)$;

(iv) the group of all invertible elements $(\mathbb{R}^*, \cdot)$.