Course 1213 - Introduction to group theory 2018

S h e e t 3

Due: at the end of the tutorial

Exercise 1

Which sets S with given operations are groups:

  (i) $S = \{-1, 0, 1, 2\}$ with respect to addition;

 (ii) $S = \mathbb{Z} \setminus \{0\}$ with respect to multiplication;

(iii) $S = \{3n : n \in \mathbb{Z}\}$ with respect to addition;

(iv) $S = \{3n : n \in \mathbb{Z}\}$ with respect to multiplication;

 (v) $S = \mathbb{Z}$ with respect to subtraction;

(vi) $S = \{5^n : n \in \mathbb{Z}\}$ with respect to multiplication.

Exercise 2

Prove or disprove that in any group

  (i) identity e is the only solution of the equation $x^2 = x$.

 (ii) identity e is the solution of $x^3 = x$?

Exercise 3

For $a = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 3 & 2 & 5 \end{pmatrix}$ and $b = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 1 & 2 & 4 \end{pmatrix}$, compute ab and a^{-1};