September 24, 2018 Lecturer Dmitri Zaitsev

Course 1213 - Introduction to group theory 2018

Sheet 2

Due: at the end of the tutorial

Exercise 1

Let $f\colon S \to T$ be a map and $A, B \subset S$ be two subsets.

(i) Show that $f(A \cap B) \subset f(A) \cap f(B)$;

(ii) Show that $f(A \setminus B) \supset f(A) \setminus f(B)$ and illustrate by example that "$\supset$" cannot be replaced by "$=$" in general.

Exercise 2

How many maps, injective maps, surjective maps and bijective maps f from A to B exist for

(i) $A = \{0\}$, $B = \{1, 2\}$;

(ii) $A = \{1, 2\}$, $B = \{0\}$;

(iii) $A = \{1, 2, 3\}$, $B = \{1, -1\}$.

Exercise 3

Which binary operations $*$ on the natural numbers $\mathbb{N}$ are commutative and which are associative:

(i) $m * n = m + n + 1$;

(ii) $m * n = \frac{mn}{2}$;

(ii) $m * n = 1$.

Exercise 4

For which binary operations $*$ on the rational numbers $\mathbb{Q}$ there is an identity element:

(i) $m * n = mn$;

(ii) $m * n = m + n + 1$;

(iii) $m * n = \frac{m+n}{3}$;

(iv) $m * n = -1$.

Exercise 5

Prove that associativity $(ab)c = a(bc)$ holds automatically whenever one of the elements a, b, c is the identity e.