December 4, 2016 Lecturer Dmitri Zaitsev

Course 1213 - Introduction to group theory 2016

S h e e t 9

Due: at the end of the tutorial

Exercise 1

Find all solutions of the system in $z \in \mathbb{Z}$:

(i)

$$\begin{cases} z \equiv 1 \bmod 5 \\ z \equiv 4 \bmod 6 \\ z \equiv 5 \bmod 8 \end{cases},$$

(ii)

$$\begin{cases} z \equiv 1 \bmod 5 \\ z \equiv -1 \bmod 3 \\ z \equiv 5 \bmod 2 \end{cases}.$$

Exercise 2

Consider the action of $G := \mathbb{Z}$ on $S := \mathbb{Z}_2 \times \mathbb{Z}_4$ given by

$$z \cdot ([a], [b]) := ([z + a], [z + b]).$$

(i) Show that this indeed is a well-defined group action.

(iii) What are the orbits $G \cdot (a, b)$ and what are the stabilizers $G_{(a,b)}$ for $(a, b) \in S$? Are these groups cyclic?

Exercise 3

A group action $G \times S \to S$ is called *transitive* if for every $x, y \in S$, there exists $g \in G$ with $g \cdot x = y$. In the following assume $|S| > 1$, i.e. S has more than one element. Prove or disprove:

(i) There exists a transitive group action on a set S, whose stabilizers satisfy $G_x \neq \{e\}$ for all $x \in S$.

(ii) For every group $G \neq \{e\}$, there exists a set S with $|S| > 1$ and an transitive action of G on S such that $G_x \neq \{e\}$ for all $x \in S$.

(iii) If G is finite and $G_x \neq \{e\}$ for all $x \in S$, then all orbits $G \cdot x$ have the same number of elements.