November 28, 2016 Lecturer Dmitri Zaitsev

Course 1213 - Introduction to group theory 2016

S h e e t 8

Due: at the end of the tutorial

Exercise 1

(i) For any groups G_1 and G_2 prove that $G_1 \times \{e\}$ and $\{e\} \times G_2$ are normal subgroups in $G_1 \times G_2$.

(ii) Show that the quotient group $(G_1 \times G_2)/(G_1 \times \{e\})$ is isomorphic to G_2.

(iii) Prove that the intersection of any family of normal subgroups in a group is again a normal subgroup.

Exercise 2

Determine the order of each of the following quotient groups:

(i) $\mathbb{Z}_8/\langle[2]\rangle$

(ii) $\mathbb{Z}_8/\langle[3]\rangle$.

Exercise 3

Let H be a normal subgroup in a group G, and $f\colon G \to G'$ a group homomorphism. Show that the construction

$$[g] \mapsto f(g)$$

gives a well-defined map $G/H \to G'$ if and only if $H \subset \ker f$.

Exercise 4

Use Exercise 3 to find all integers m such that $z \mapsto mz$ gives a well-defined map

$$f_m\colon \mathbb{Z}_{12} \to \mathbb{Z}_{12},$$

and determine the image and the kernel for each such f_m.